**Title page**  
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Preface  
This report is the result of a student project that is part of the educational program of the Bachelor Knowledge Engineering at Maastricht University. As such its purpose is in part to comprise the results of said project but in addition can give insight into possible approaches to solving knapsack problems. Furthermore it presents the results of experiments involving several such algorithmic approaches and judges their performance.

Summary

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# 1. Introduction

## 1.1 Assignment description

The assignment for the project on which this report is based was to build a computer application with a user friendly interface that can be used for solving so-called three dimensional knapsack problems.

The assumptions are that a company owns trucks with a cargo space of 16.5 m long, 2.5 m wide and 4.0 m high and that it transports parcels of three different types: A, B and C. The sizes of the types are:  
  
 A: 1.0 x 1.0 x 2.0  
 B: 1.0 x 1.5 x 2.0  
 c: 1.5 x 1.5 x1.5  
  
A parcel of a given type also has a certain value, denoted by vA, vB and vC for types A, B and C respectively. The computer application should compute, for a given set of parcels (that may or may not fit into a truck), a packing that maximises the total value.

The application does not have to find the best answer in all cases, but it should be able to find a good approximation. The application should also make a 3D-visualisation of its answers – from different perspectives.  
  
The application should be used to answer the following questions (see 3.1 and 3.2):

1. Is it possible to fill the complete cargo space with A, B and/or C parcels, without having any gaps?
2. If parcels of type A,B and C represent values of 3, 4 and 5 units respectively, then what is the maximum value that can be stored in the cargo space?

In addition, after answering the two questions above, it should now be assumed that the company transports pentomino shaped parcels of types L, P and T (see Appendix A, Figure 1), where each of these pentominoes consists of 5 cubes of size 0.5 x 0.5 x 0.5. For those assumptions the following questions were posed:

1. Is it possible to fill the complete cargo space with L, P and/or T parcels without having any gaps?
2. If parcels of type L, P and T represent values of 3, 4 and 5 units respectively, then what is the maximum value that can be stored in the cargo space?

Beyond all of the tasks above we were advised to conduct experiments of our own once we had one or multiple functioning algorithms.

## 1.2 Problem definition

All tasks given in the project assignment deal with the optimisation of a packing of certain types of packages in a constricted three-dimensional space. Since the main purpose of the application is to optimise the total value of a packing while also fitting all the included packages in the given space, the problem at hand could be defined as a three-dimensional knapsack problem. With the constraints being the dimensions in which all of the packages included in the solution had to fit without overlapping, the main goal of the algorithm that should solve the problem was to maximise the total value of the included packages while adhering to the constraints.

While similar kinds of optimisation or knapsack problems can occur in a wide variety of fields and similar algorithmic approaches to the ones chosen in our own project may be applicable, the problem at hand in particular is focused on the packing of a three-dimensional space. As such the algorithms developed during our research are fit to optimally fill a cargo space of a truck (as is the context of the assignment) or any similar sort of container.

## 1.3 Structure

Chapter 2 of the report describes the three algorithmic approaches to solve the assigned problem (a greedy approximation algorithm, a hill climbing and a genetic algorithm) as well as some aspects of their implementation in the application that was the result of this project. Chapter 3 gives concise answers to the four individual questions posed by the project assignment (see 1.1) without going into a lot of detail regarding the implication of the results. In chapter 4 several experiments are described in which certain parameters crucial for the performance of the three chosen algorithms are varied, including their results. The first part of the chapter deals with the variation of aspects of the problem that are applicable to all algorithms, the later parts describe experiments on individual aspects of each algorithm. Lastly, in chapter 5, we draw conclusions from the previously described results of our experiments.

# 2. Algorithms for the knapsack problem

## 2.1 Greedy approximation algorithm

While not in the form of a three-dimensional knapsack problem such as the one that is subject of this project report, the idea of a so called greedy approximation algorithm stems from the American mathematical scientist George Dantzig. In his version of the algorithm the items (in this case packages) to be placed in the knapsack are sorted by their value per weight (which is the volume for this problem) and then placed in the knapsack in the resulting sequence.

The basic implementation of that principle in our algorithm is the following. From the packages that are chosen by the user to be placed in the cargo space the ones with the highest value to volume ratio are placed first as long as there is a supply of them. When the supply of packages of the first type is exhausted and there is empty space left, the next type of package will be placed. That process is repeated until all packages have been placed or none of the packages left can be placed anymore.

The placement method employed in the application tries to place a new package in the top right front corner of the cargo space. (If the package overlaps with a different package in that initial position, it is not considered for any other placement anymore and the algorithm will attempt to place the next package.) From that initial position the package is first moved as far back, then as far left and finally as far down in the cargo space as possible (corresponding to movements along the y-axis, x-axis and z-axis; see Figure 1).



Figure 1

## 2.2 Hill climbing algorithm

The hill-climbing search algorithm is simply a loop that continually moves in the direction of increasing value, that is, uphill. It terminates when no successor has a higher value.  
The algorithm starts with an arbitrary solution to a problem and attempts to find a better solution by incrementally changing a single element of the solution. If the change produce a better solution the starting solution get replaced by the new solution, repeating until no further improvements can be found.

**function** HILL-CLIMBING(problem) **returns** a state that is a local maximum

current ← MAKE-NODE(problem.INITIAL-STATE)

**loop do**

neighbor ← a highest-valued successor of current

**if** neighbor.VALUE ≤ current.VALUE **then return** current.STATE

current ← neighbor

For the hill-climbing algorithm used to maximize the total value of our cargo, we start the algorithm by finding an arbitrary solution that tries to fill our cargo with random parcels.   
From this starting solution we create n neighbour solutions by changing a random package in the cargo and trying to fill the remaining empty space with random packages.   
After we created the neighbours a method will chose the best next solution based on an objective function that aims to maximize the total value of our cargo.

For testing purposes we might decide to either allow rotations of the parcels or not. Also we can try starting the algorithm with a arbitrary solution found by the previous developed greedy algorithm.

## 2.3 Genetic algorithm

The third algorithm implemented in the program is a genetic algorithm. To give a short explanation for readers without any prior knowledge about the principle of genetic algorithms, it can be said that such algorithms imitate Natural Selection. This requires an appropriate encoding of the “genetic” information that defines an individual’s characteristics and thus its fitness. In addition methods for reproduction (involving two parent individuals and a resulting child individual) and mutation have to be provided, two key elements in the evolution of the population of individuals. Furthermore a method to select the individuals allowed to reproduce needs to be implemented. This selection method is usually based on the fitness of individuals for which a fitness evalutation method has to be chosen.

For the genetic algorithm implemented for this project a binary encoding of the “genetic” information of individuals was chosen. The chromosomes of each individual are represented by an array of 0 and 1 values, which are generated randomly for the initial population. Each of these “genes” decides whether a certain package in a certain state of rotation and a certain position in the cargo space is included in the solution. To evaluate the fitness of an individual resulting from that information (i.e. the actual packing in the cargo space), this binary string of numbers is interpreted and converted into a packing. This packing is then evaluated to calculate the fitness of that individual.

For the purpose of experimenting with the effects of changing vital methods and parameters on the performance of the genetic algorithm, several different selection methods and fitness evaluation functions have been implemented.

# 3. Assignment results

## 3.1 Using rectangular packages

Considering the results achieved so far, using mostly the greedy approximation algorithm, there is no definitive solution to the first question posed. While it might be possible to fill the entire cargo space (especially using more sophisticated algorithms), it could not be confirmed that it is possible.

As for the best result achieved so by any of the algorithms so far in terms of maximising the value of a single packing, the highest value achieved over X test runs was Y (just placeholders for now, no real tests done yet).

## 3.2 Using pentomino-shaped packages

# 4. Experiments and results

## 4.1 Principles of evaluation

### 4.1.1 Measures of performance

The most important factor taken into consideration during the evaluation of an algorithm’s performance is its ability to either maximise the value of a packing or minimise the number of gaps left in the cargo space, depending on the desired result.

Additionally the amount of time it takes the algorithm to compute a solution is considered as well since it has relevance in terms of practicability as an application for actual use. It also gives some insight into the complexity/amount of computations that the algorithm requires to find a solution.

### 4.1.2 Comparison between results

In order to be able to compare individual results, whether between those of different algorithms or those achieved

## 4.2 For all algorithms

### 4.2.1 Package type diversity

### 4.2.2 Package and container size

## 4.3 Greedy algorithm

### 4.3.1 Different methods of selection order

### 4.3.2 Rotation

### 4.3.3 Finding and filling empty space

## 4.4 Hill climbing algorithm

### 4.4.1 Varying neighbourhoods

## 4.5 Genetic algorithm

### 4.5.1 Different generation sizes

### 4.5.2 Different selection methods

### 4.5.3 Different fitness evaluation

# 5. Conclusions

## 5.1 Greedy algorithm

## 5.2 Hill climbing algorithm

## 5.3 Genetic algorithm

## 5.4 Comparison between algorithms

# Appendix A Experiment results

## A.1 Greedy algorithm

## A.2 Hill climbing algorithm

## A.3 Genetic algorithm