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# 1. Introduction

## 1.1 Assignment description

The assignment for the project on which this report is based was to build a computer application with a user friendly interface that can be used for solving so-called three dimensional knapsack problems.

The assumptions were that a company owns trucks with a cargo space of 16.5 m long, 2.5 m wide and 4.0 m high and that it transports parcels of three different types: A, B and C. The sizes of the types are:  
  
 A: 1.0 x 1.0 x 2.0  
 B: 1.0 x 1.5 x 2.0  
 c: 1.5 x 1.5 x1.5  
  
A parcel of a given type also has a certain value, denoted by vA, vB and vC for types A, B and C respectively. Our computer application should compute, for a given set of parcels (that may or may not fit into a truck), a packing that maximises the total value.

The application does not have to find the best answer in all cases, but it should be able to find a good approximation. The application should also make a 3D-visualisation of its answers – from different perspectives.  
  
The application should be used to answer the following questions (see 3.1 and 3.2):

1. Is it possible to fill the complete cargo space with A, B and/or C parcels, without having any gaps?
2. If parcels of type A,B and C represent values of 3, 4 and 5 units respectively, then what is the maximum value that can be stored in the cargo space?

In addition, after answering the two questions above, we should now assume that the company transports pentomino shaped parcels of types L, P and T (see Appendix A, Figure 1), where each of these pentominoes consists of 5 cubes of size 0.5 x 0.5 x 0.5. For those assumptions the following questions were posed:

1. Is it possible to fill the complete cargo space with L, P and/or T parcels without having any gaps?
2. If parcels of type L, P and T represent values of 3, 4 and 5 units respectively, then what is the maximum value that can be stored in the cargo space?

Beyond all of the tasks above we were advised to conduct experiments of our own once we had one or multiple functioning algorithms.

## 1.2 Problem definition

All tasks given in the project assignment deal with the optimisation of a packing of certain types of packages in a constricted three-dimensional space. Since the main purpose of the application to be built is was to optimise the total value of a packing while also fitting all the included packages in the given space, the problem at hand could be defined as a three-dimensional knapsack problem. With the constraints being the dimensions in which all of the packages included in the solution had to fit without overlapping, the main goal of the algorithm that should solve the problem was to maximise the total value of the included packages while adhering to the constraints.

While similar kinds of optimisation or knapsack problems can occur in a wide variety of fields and similar algorithmic approaches to the ones chosen in our own project may be applicable, the problem at hand in particular is focused on the packing of a three-dimensional space. As such the algorithms developed during our research are fit to optimally fill a cargo space of a truck (as is the context of the assignment) or any similar sort of container.

## 1.3 Structure

Chapter 2 of the report describes the three algorithmic approaches to solve the assigned problem (a greedy approximation algorithm, a hill climbing and a genetic algorithm) as well as some aspects of their implementation in the application that was the result of this project. Chapter 3 gives concise answers to the four individual questions posed by the project assignment (see 1.1) without going into a lot of detail regarding the implication of the results. In chapter 4 several experiments are described in which certain parameters crucial for the performance of the three chosen algorithms are varied, including their results. The first part of the chapter deals with the variation of aspects of the problem that are applicable to all algorithms, the later parts describe experiments on individual aspects of each algorithm. Lastly, in chapter 5, we draw conclusions from the previously described results of our experiments.

# 2. Algorithms for the knapsack problem

## 2.1 Greedy approximation algorithm

While not in the form of a three-dimensional knapsack problem such as the one that is subject of this project report, the idea of a so called greedy approximation algorithm stems from the American mathematical scientist George Dantzig. In his version of the algorithm the items (in this case packages) to be placed in the knapsack are sorted by their value per weight (which is the volume for this problem) and then placed in the knapsack in the resulting sequence.

The basic implementation of that principle in our algorithm is the following. From the packages that are chosen by the user to be placed in the cargo space the ones with the highest value to volume ratio are placed first as long as there is a supply of them. When the supply of packages of the first type is exhausted and there is empty space left, the next type of package will be placed. That process is repeated until all packages have been placed or none of the package left can be placed anymore.

The (current) placement method employed (which could be varied) first tries to place a new package in the top right front corner of the cargo space. (If the package overlaps with a different package in that initial position, it is not considered for any other placement anymore and the algorithm will attempt to place the next package.) From that initial position the package is first moved as far back, then as far left and finally as far down in the cargo space as possible (corresponding to movements along the y-axis, x-axis and z-axis; see Appendix A, Figure 2).

## 2.2 Hill climbing algorithm

## 2.3 Genetic algorithm

# 3. Assignment results

## 3.1 Using rectangular packages

## 3.2 Using pentomino-shaped packages

# 4. Experiments and results

## 4.1 Principles of evaluation

### 4.1.1 Measures of performance

### 4.1.2 Comparison between results

## 4.2 For all algorithms

### 4.2.1 Package type diversity

### 4.2.2 Package and container size

## 4.3 Greedy algorithm

### 4.3.1 Different methods of selection order

### 4.3.2 Rotation

### 4.3.3 Finding and filling empty space

## 4.4 Hill climbing algorithm

### 4.4.1 Varying neighbourhoods

## 4.5 Genetic algorithm

### 4.5.1 Different generation sizes

### 4.5.2 Different selection methods

### 4.5.3 Different fitness evaluation

# 5. Conclusions

## 5.1 Separate for single algorithms

### 5.1.1 Greedy algorithm

### 5.1.2 Hill climbing algorithm

### 5.1.3 Genetic algorithm

## 5.2 Comparison between algorithms

# Appendix A Figures

# Appendix B Experiment results

## B.1 Greedy algorithm

## B.2 Hill climbing algorithm

## B.3 Genetic algorithm